Electric Power: An Example of the Derivative of a Product

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Introduction

- Electric power is the rate at which electric energy is transferred by an electric circuit.
- The unit of power is the Watt, which is one joule per second.
- Electric power that is used in the home usually comes from one of two sources.

1. Electric generators (operated by a power company)
2. Batteries

Mathematical Definition of Power

- $P(t) = v(t)\, i(t)$

  where $v(t)$ is electric potential or voltage measured in volts,

  and $i(t)$ is electric current measured in amperes.

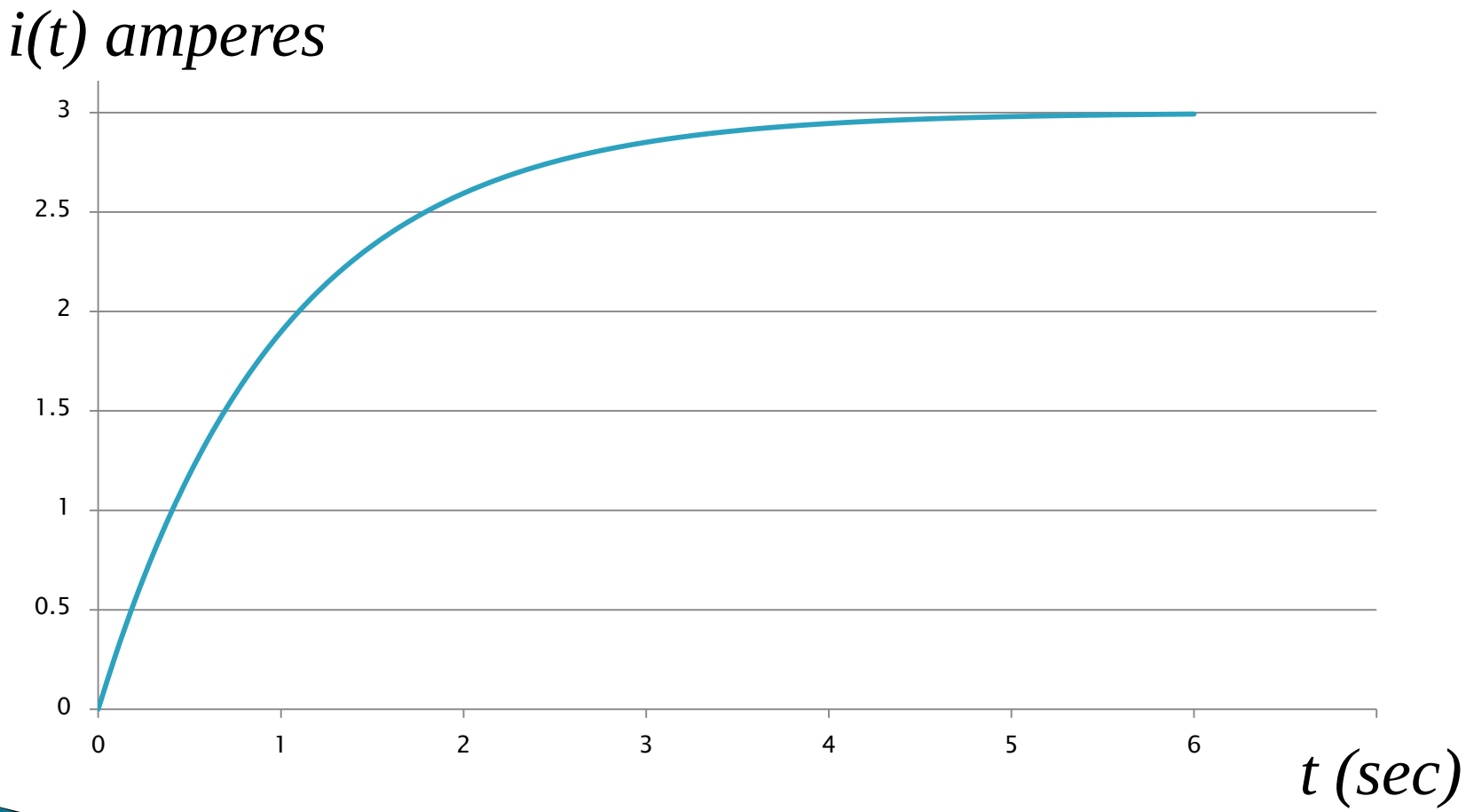
The current is measured and its values are tabulated as below.

t	i(t)
0	0
0.1	0.285488
0.2	0.543808
0.3	0.777545
0.4	0.98904
0.5	1.180408
0.6	1.353565
0.7	1.510244
0.8	1.652013
0.9	1.780291
1	1.896362
1.1	2.001387
1.2	2.096417
1.3	2.182405
1.4	2.260209
1.5	2.33061
1.6	2.39431
1.7	2.451949
1.8	2.504103
1.9	2.551294
2	2.593994

t	i(t)
2.1	2.632631
2.2	2.667591
2.3	2.699223
2.4	2.727846
2.5	2.753745
2.6	2.777179
2.7	2.798383
2.8	2.81757
2.9	2.83493
3	2.850639
3.1	2.864852
3.2	2.877713
3.3	2.88935
3.4	2.89988
3.5	2.909408
3.6	2.918029
3.7	2.925829
3.8	2.932888
3.9	2.939274
4	2.945053

t	i(t)
4.1	2.950282
4.2	2.955013
4.3	2.959294
4.4	2.963168
4.5	2.966673
4.6	2.969844
4.7	2.972714
4.8	2.975311
4.9	2.97766
5	2.979786
5.1	2.98171
5.2	2.98345
5.3	2.985025
5.4	2.98645
5.5	2.98774
5.6	2.988906
5.7	2.989962
5.8	2.990917
5.9	2.991782
6	2.992564

Plot of the Current



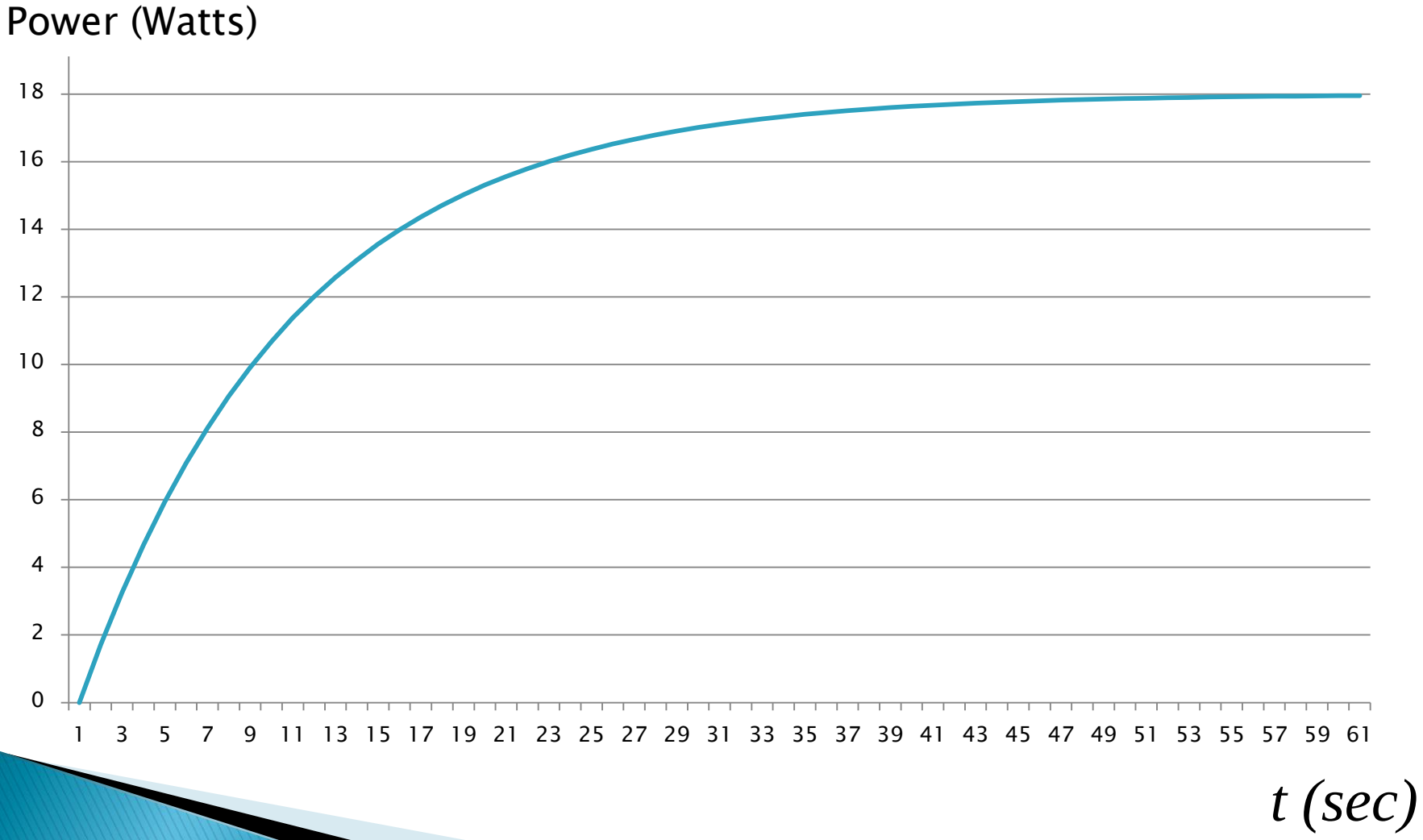
DC Voltage

- A Direct Current voltage is one whose electric potential does not vary with time.
- For this example, let's assume that the voltage $v(t) = 6$ volts

We can obtain values of the power by multiplication, $P(t) = i(t)*6$

t	i(t)	P(t)
0	0	0
0.1	0.285488	1.712926
0.2	0.543808	3.262846
0.3	0.777545	4.665272
0.4	0.98904	5.934239
0.5	1.180408	7.082448
0.6	1.353565	8.121391
0.7	1.510244	9.061465
0.8	1.652013	9.912079
0.9	1.780291	10.68175
1	1.896362	11.37817
1.1	2.001387	12.00832
1.2	2.096417	12.5785
1.3	2.182405	13.09443
1.4	2.260209	13.56125
1.5	2.33061	13.98366
1.6	2.39431	14.36586
1.7	2.451949	14.7117
1.8	2.504103	15.02462
1.9	2.551294	15.30776
2	2.593994	15.56396
2.1	2.632631	15.79578
2.2	2.667591	16.00554
2.3	2.699223	16.19534
2.4	2.727846	16.36708
2.5	2.753745	16.52247
2.6	2.777179	16.66308
2.7	2.798383	16.7903
2.8	2.81757	16.90542
2.9	2.83493	17.00958
3	2.850639	17.10383
3.1	2.864852	17.18911
3.2	2.877713	17.26628
3.3	2.88935	17.3361
3.4	2.89988	17.39928
3.5	2.909408	17.45645
3.6	2.918029	17.50817
3.7	2.925829	17.55498
3.8	2.932888	17.59733
3.9	2.939274	17.63565
4	2.945053	17.67032
4.1	2.950282	17.70169
4.2	2.955013	17.73008
4.3	2.959294	17.75577
4.4	2.963168	17.77901
4.5	2.966673	17.80004
4.6	2.969844	17.81907
4.7	2.972714	17.83629
4.8	2.975311	17.85186
4.9	2.97766	17.86596
5	2.979786	17.87872
5.1	2.98171	17.89026
5.2	2.98345	17.9007
5.3	2.985025	17.91015
5.4	2.98645	17.9187
5.5	2.98774	17.92644
5.6	2.988906	17.93344
5.7	2.989962	17.93977
5.8	2.990917	17.9455
5.9	2.991782	17.95069
6	2.992564	17.95538

Plot of the Power



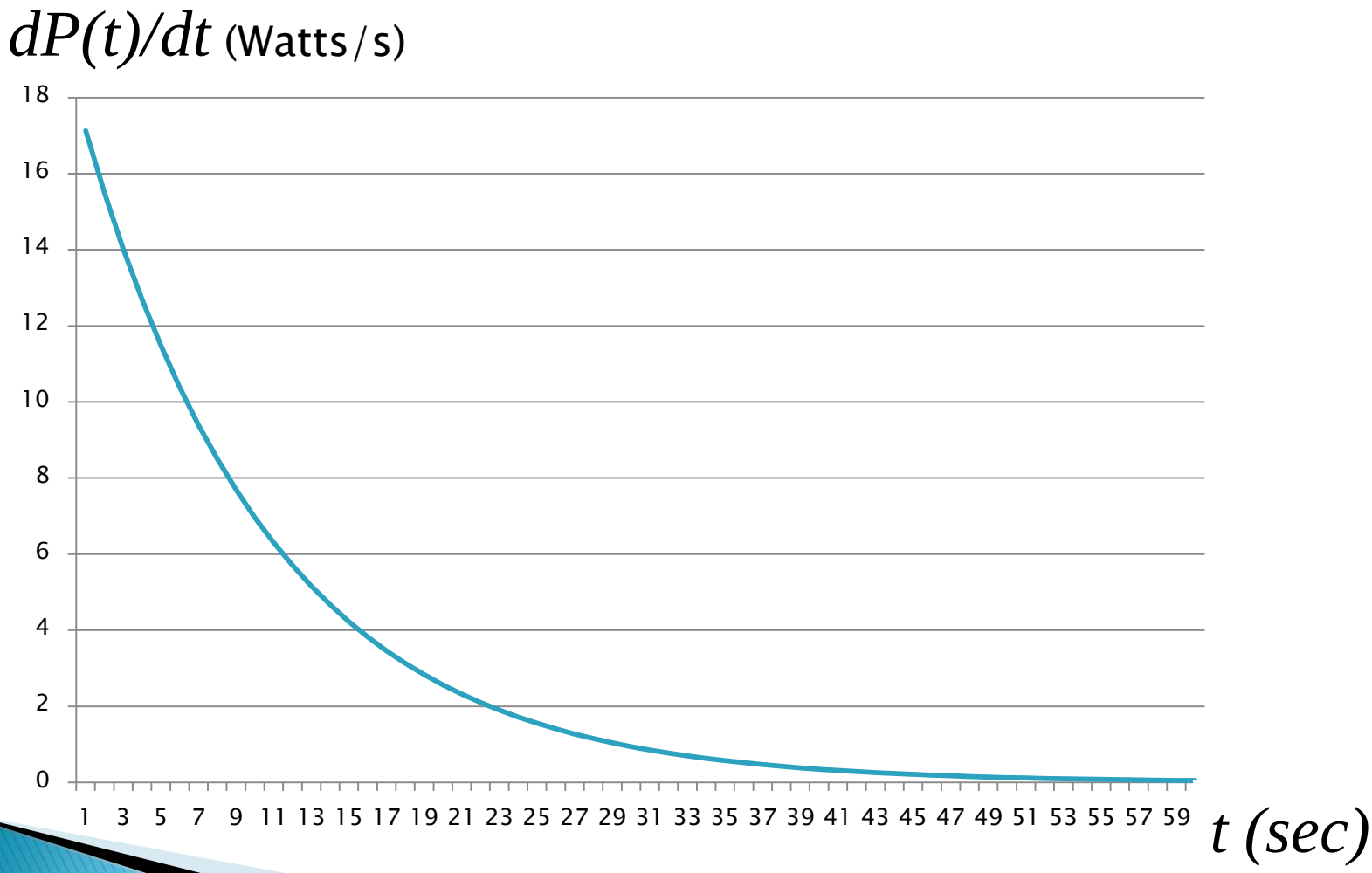
Estimating the Derivative of the Power

- We can estimate the power by differencing adjacent values of the power and dividing by the time difference.

Table to Estimate the Derivative of Power

t	P(t)	dP(t)/dt
0	0	17.12926
0.1	1.712926	15.4992
0.2	3.262846	14.02426
0.3	4.665272	12.68967
0.4	5.934239	11.48209
0.5	7.082448	10.38942
0.6	8.121391	9.40074
0.7	9.061465	8.506141
0.8	9.912079	7.696675
0.9	10.68175	6.964239
1	11.37817	6.301504
1.1	12.00832	5.701837
1.2	12.5785	5.159235
1.3	13.09443	4.668269
1.4	13.56125	4.224025
1.5	13.98366	3.822056
1.6	14.36586	3.458339
1.7	14.7117	3.129234
1.8	15.02462	2.831448
1.9	15.30776	2.562
2	15.56396	2.318194
2.1	15.79578	2.097589
2.2	16.00554	1.897977
2.3	16.19534	1.71736
2.4	16.36708	1.553932
2.5	16.52247	1.406056
2.6	16.66308	1.272252
2.7	16.7903	1.151181
2.8	16.90542	1.041632
2.9	17.00958	0.942507
3	17.10383	0.852816
3.1	17.18911	0.77166
3.2	17.26628	0.698227
3.3	17.3361	0.631782
3.4	17.39928	0.57166
3.5	17.45645	0.517259
3.6	17.50817	0.468035
3.7	17.55498	0.423496
3.8	17.59733	0.383195
3.9	17.63565	0.346729
4	17.67032	0.313733
4.1	17.70169	0.283878
4.2	17.73008	0.256863
4.3	17.75577	0.232419
4.4	17.77901	0.210302
4.5	17.80004	0.190289
4.6	17.81907	0.172181
4.7	17.83629	0.155795
4.8	17.85186	0.14097
4.9	17.86596	0.127554
5	17.87872	0.115416
5.1	17.89026	0.104433
5.2	17.9007	0.094495
5.3	17.91015	0.085502
5.4	17.9187	0.077366
5.5	17.92644	0.070003
5.6	17.93344	0.063342
5.7	17.93977	0.057314
5.8	17.9455	0.05186
5.9	17.95069	0.046925
6		

Plot of the Estimated Derivative of Power



Use the Product Rule to compute the derivative function

$$P(t) = v(t)\, i(t)$$

$$dP(t)/dt = (dv(t)/dt)\ \ i(t) + v(t)\, di(t)/dt$$

Let $v(t) = 6$ and $i(t) = 3 - 3\, e^{-t}$

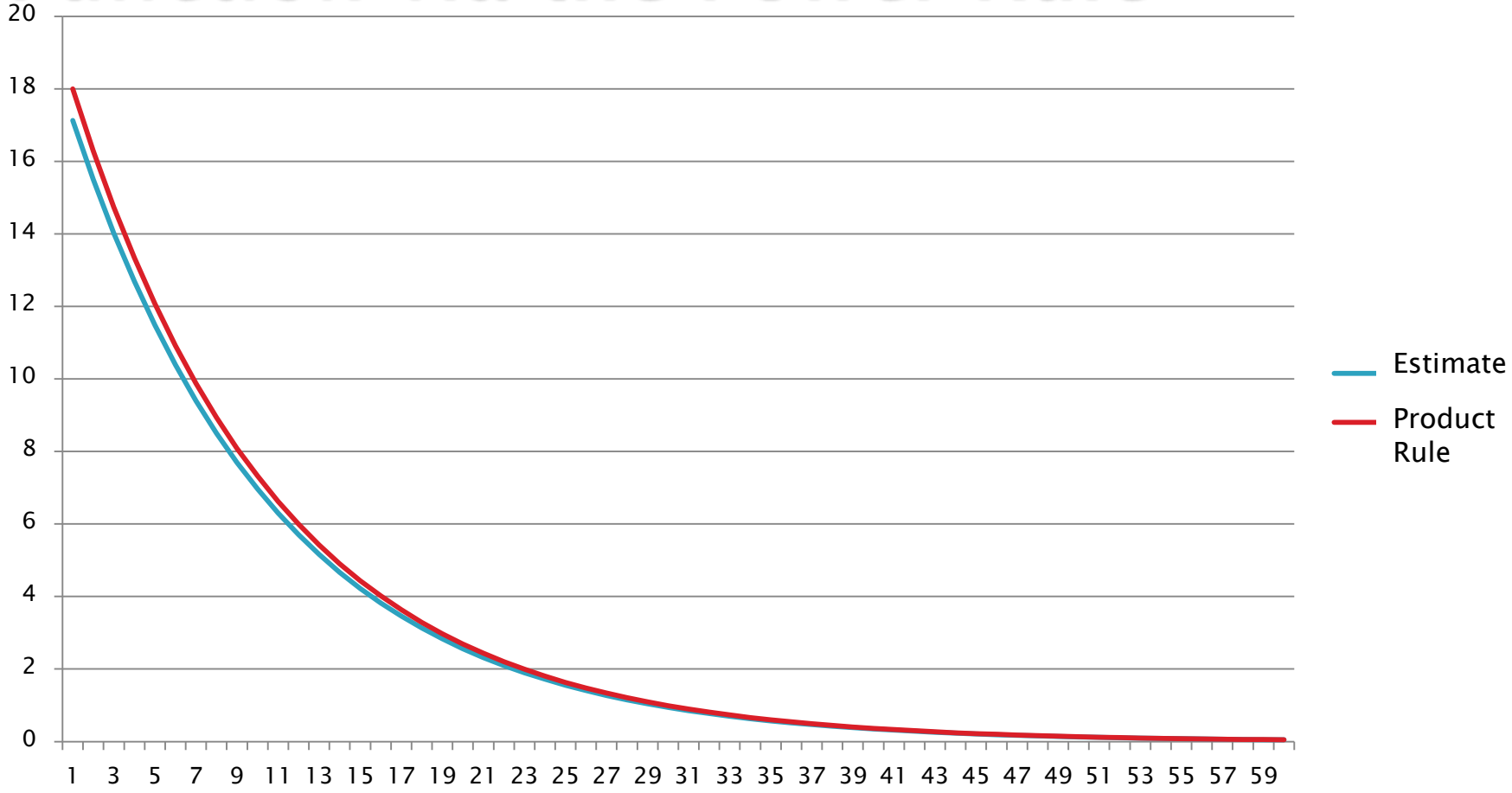
Function used to generate the table of data.

$$dv(t)/dt = 0,\ di(t)/dt = -3\,(-1)\, e^{-t} = 3\, e^{-t}$$

So, $dP(t)/dt = (dv(t)/dt)\, i(t) + v(t)\, di(t)/dt$

$$= 0 + 6\,(\,3\, e^{-t}) = \boxed{18\, e^{-t}}$$

Comparison of the Estimated Derivative of Power Vs. Derivative Function via the Power Rule



Comments

- The Product Rule allows us to obtain a closed form expression for the derivative of a function.
- Comparison of the estimated derivative with that obtained from the closed form found via the Product Rule are quite similar.